SYNOPSIS

The thesis entitled "NUMERICAL SOLUTION OF SOME IMPORTANT PHENOMENON ARISING IN FLUID FLOW THROUGH POROUS MEDIA" is an attempt for single as well as double phase immiscible fluid flow problems through porous media, traditionally described by non-linear partial differential equation.

It consists of 7 chapters. Chapter-1 is an introduction of the subject matter and physical & mathematical background for understanding of investigated problems given in subsequent 6 chapters. Chapter-2 gives the basics of similarity Analysis and Differential Quadrature Method.

The Chapters 3 to 7 are devoted for seepage problem, ground water recharge in unsaturated porous media, longitudinal dispersion, fingering phenomenon, imbibition phenomenon. The problems discussed in the thesis have great importance in some diversified fields such as mechanics, water purification, petroleum engineering, ceramic engineering, ground water hydrology etc. The large amounts of literature of these branches of engineering and science have contributed much on the subject. An opportunity of hydrodynamics in porous media allows for division into several parts.

The chapter-1 gives the introduction of the flow through porous media & it's applications in the various fields of science & engineering. The historical overview of the development of this field of research is also included. It also provides the recent advances & contemporary interest in the subject.

It also contains the physico-mathematical background of the subject. It discusses the basic Darcy's law governing the flow through porous media, its validity & extension for the flows in homogeneous and heterogeneous media. The author has introduced the fundamental definitions, some basic flow

equations, namely, the equation of continuity applied to each phase individually and the relation between capillary pressure and phase saturation which are sufficient for a complete specification of the flow system. Further author has also discussed the relative permeability, the pressure discontinuity of the phase, the fingering phenomenon, the imbibition phenomenon.

The chapter-2 represents the mathematical background of similarity analysis and Differential Quadrature Method. The method to search particular type of transformations that transforms the partial differential equation into an ordinary differential equation is known as similarity method. The concept of mathematical similarity was first introduced by Helmoltz through the dimensional analysis approach. Later on importance of similarity parameters in physical sciences was demonstrated by Reynolds, Backingham, Blasius & Sedov and they developed the concept of similarity in more meaningful way till Morgan, Birkhoff showed that the boundary layer equations were invariant under certain group of transformations which reduce the number of independent variables by one. The power of various similarity techniques is recently demonstrated by various authors like Hansen, Roger & Ames, Bluman & Kumei.

Among the various similarity methods like separation of variables, free parameters, dimensional analysis, group theoretic technique etc, the group theoretic technique is more useful. Using this group theoretic method we have discussed the fingering phenomenon of the porous media to convert non-linear partial differential equation into an ordinary differential equation. We then apply Differential Quadrature Method to this Ordinary Differential Equation, which is explaining in chapter-6.

The Differential Quadrature Method (DQM) is a numerical solution technique for finding the numerical solution of initial and/or boundary value problems. It was developed by the late Richard Bellmann and his associates in the early 70s & since then the technique has been successfully employed in a

variety of problems in engineering and physical sciences. The DQ method, similar to the conventional integral quadrature method, approximates the derivative of a function at any location by a linear summation of all the functional values along a mesh (grid) line. The key procedure in the DQ method and its applications were rapidly developed after the late 1980s, thanks to the innovative work in the computation of the weighting coefficients by other researchers. The text book of Shu (2000) represents the first comprehensive work on the DQ Method & its applications.

DQM discretized the space derivatives giving a system of ordinary differential equations with respect to time and the fourth order Runge Kutta Method (RKM-4) is employed for solving this system. Stabilities of the system of ordinary differential equations and RKM-4 are considered and step sizes are arranged accordingly. The solutions are presented in terms of table values comparing with the exact solution. This method exhibits high accuracy and efficiency with minimal computations efforts comparing to the other numerical methods (Bert,Malik). Thus combination of DQM and 4^{th} order RK method give very good numerical technique for solving porous media problems.

The chapter-3 discusses a seepage problem of groundwater down a sloping bedrock in heterogeneous as well as in homogeneous soil with vertical direction by utilizing the hydraulic theory. The seepage of ground water in two-layered soil with an inclined boundary when lower layer is heterogeneous and upper one is homogeneous. The equation of free surface is determined by method of multiple scaling techniques. The numerical solution of this equation is obtained by DQM. The equation is reduced into a system of ordinary differential equations by DQM. The obtained system of ordinary differential equations is then solved by a fourth-stage RK scheme.

The chapter-4 represents one dimensional vertical ground water recharge problems through porous media have been discussed under certain standard available assumptions, that the permeability of the medium as specific function of moisture content which yields a non-linear partial differential equation which is solved by Differential Quadrature Method to convert it into system of ordinary differential equation are then solved by the fourth stage RK scheme. Such problem has great importance in many branches of engineering, such as civil, hydrology, sanitary and irrigation.

The chapter-5 deals with the discussion of miscible displacement. This type of displacement has been of great concern to hydrologists who have been studying the problem of displacement of fresh water by sea water in coastal areas. The oil industry has also become involved in miscible displacement studies in connection with the possibility of flushing oil by solvents from reservoirs. In spite of the importance of miscible displacement, its study is still in its infancy. A series of theories has been proposed for various types of flow. This chapter, in particular, discusses the phenomenon of longitudinal dispersion which is the process by which miscible fluids in laminar flow mix in the direction of the flow. The phenomenon is discussed by regarding the cross-sectional flow velocity as time dependent in a specific form. The mathematical formulation of the phenomenon yields a non-linear partial differential equation. The equation is reduced into a system of ordinary differential equations by DQM. The obtained system of ordinary differential equations is then solved by a fourth-stage RK scheme.

The chapter-6 concerns with the phenomenon of fingering (instability) in displacement process involving two immiscible fluids (oil & water) through the homogeneous porous media with mean capillary pressure. It is well known physical fact that when a fluid contained in a porous media is displaced by another fluid of lesser viscosity, instead of regular displacement of the whole

front protuberances may occur which shoot through the porous media at relatively great speed. This phenomenon of the occurrence of protuberance is known as fingering. In the statistical treatment of fingering only average cross sectional area occupied by the fingers was observed while the size & shape of the individual fingers are neglected. In the present chapter, author had discussed the phenomenon of fingering with mean pressure in the finger. This physical phenomenon is represented by the PDE. This PDE is then converted to ordinary differential equation using similarity analysis technique and this ODE is solved by DQ Method. Numerical values are shown by the table.

The chapter-7 discusses mathematically the phenomenon of imbibition in double phase flow of two immiscible fluids in a homogeneous porous medium with capillary pressure. The imbibition phenomenon arises due to the difference in the wetting abilities of the phases. It is assumed that the injection of a preferentially wetting fluid into a porous medium saturated with another non-wetting fluid is initiated by imbibition and the injected fluid phases of different wetting abilities with respect to the porous media. By assuming the validity of Darcy's law, a mathematical formulation of the imbibition phenomenon yields a non-linear partial differential equation. The numerical solution of this PDE is obtained by DQM. The equation is reduced into a system of ordinary differential equations by DQM. The obtained system of ordinary differential equations is then solved by a fourth-stage RK scheme. The numerical values are shown in tabular form.

Conclusion:-The Differential Quadrature Method is accompained with the fourth order Runge-Kutta Method for solving the different type of porous media problems. Derivatives in space directions are discretized with DQM while time derivative is discretized by using 4th order RKM. The combination of these two methods performs very well since the resulting system of ordinary differential equations is stable for different type of boundary conditions and the

explicit 4th order RKM has quite a large stability region as well as practical to use.

Thus, to sum up, the subject matter of the preset work adds to the existing investigations by making some original contribution, giving fresh interpretation of known facts and extending the scope of mathematical study. I have obtained numerical solution of the different porous media problems. It is believed that the present work will initiate a new direction of research in fluid dynamics through porous media. Thus it is my humble attempt to add the valuable contribution of original work in this field.

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